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Weibull Modelling of the Operational Costs of a Start-Stop and Idling Traffic Drive Decision Applications

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ABSTRACT

In this study, start-stop and idling time traffic data are presented from the results of traffic time experimental investigation and are thereafter analyzed using appropriate mathematical models to generate start-stop/idling failure time data, then properly fitted with a Weibull function to effectively represent failure and survival probability distribution alongside failure rate of the assumed series connected start-stop system components (75amp flooded lead acid battery, DC starter motor and the ignition system) and the idling system components (failure of one component, is the failure of the entire system), hence the need to determine the distributions parameters, for accurate representation. A failure plus right-censored and all right-censored data sets for both drive options were presented for a Bayesian Weibull parameters estimation and subsequent operational costs analysis which reported the following costs: the starter system operating cost, C_{ss} = ₦91,763.88, that of the idling cost operation C_{is} = ₦102,517.30, with start-stop cost savings of C_S = ₦ 10753.30 or 10.49%. The operating costs for the start-up system for the all right-censored failure dataset gives; C_{ss} = ₦575.24 and that of the idling operation is C_{is}= ₦2,411.25, with a cost savings of C_S = ₦1,836.01 (76.14%) of the start-stop system over that of idling resulting from the idling system higher failure probability and fuel consumption cost. The conclusion drawn from the study is that, the start-stop traffic drive decision is more economical about the determined traffic time, although for newly installed.

Keywords — Start-Stop, Idling, Bayesian, Right-Censored, Hasting Algorithm, Normalization Constant, Posterior Probability and Failure Data.

1. INTRODUCTION

Vehicular traffic congestion is common to every region of the world, even though its intensity may vary due to properly planned and maintained road network and some sundry reasons. The main aim of controlling traffic in busy areas, road intercessions, affected areas of occurrences of natural disasters and vehicular accident, bad spots on deplorable roads and other likely traffic prone areas is to avoid traffic build up at such sections or spots , which may possibly cause traffic congestion that requires vehicle owners, whether private or commercial to wait for a period of time before passage is granted electronically or manually. The issues are worst off in developing countries with deplorable road conditions, were drivers find it difficult to access some sections of the road due to road washed off, uncompleted road project or poorly constructed community intervention projects.

The above narrative of traffic congestion mainstreams the issue of waiting time and traffic flow (passage) time in traffic control. The decision of any driver during the waiting time of the traffic hold ups is one of starts–stop or idling, Paul et al. (2015). In the start - stop decision, the driver starts the vehicle any time there is a signal to move and halts the vehicle by putting off the engine when the stop signal is issued. For the idling case, the driver decides to run the engine while the vehicle is brought to a monetary halt (running engine without vehicular motion). Either of these decision taken by a driver in a traffic waiting time, is influenced by the driver's perceived economic reasons of fuel conservations, fault development due to wear and tears leading to replacement or repair costs, competitive advantage in in-fleet management market, and many other sundry reasons whose veracity of claims may not have been technically substantiated, Paul et al., (2015) and Singhridur, et al., (2022).

Majority of the studies for environmental reasons of greenhouse gas emission, fuel conservation and fuel residue damage of engine components suggest start-stop traffic driving decisions to idling, Factsheet (www.sedhee.gov) and Singhridur et al., (2022). Others advocate that, provided restart cycles are below 10 restarts per day, then the start-stop drive decision has the potentials for the elimination of short-duration idling that may eventually cumulate to substantial costs savings, Paul et al., (2015). Based on, vehicle useful life of 120,000 miles or 10 years period, Paul et al, (2015), developed an economic model that analysed battery life on 32 miles per day travel distance at an average base line start cycle of 3 cycles per day. The potentials of failure declined in their study for an increase above the base line for the battery and the starter system.

In this study, though a costs comparison like that of Paul et al, (2015) was done, the idling system is firmly tied to the cooling system failure and replacement cost is based on the probability predictions of the cooling system failure function. Similarly, the start – stop decision cost is also based on the probability predictions of the starter system failure function. We consider the failure of any of the components part of the cooling system (radiator, water pump, coolant and fans) and the starting system (starter motor, battery and electrical system of current supply) as the failure of the entire system based on the assumption of a series system of Unlike previous studies, failure time data for the starter and idling systems are primarily data generated from traffic time experimentation and subsequent failure times tests and analysis.

2. METHODOLOGY

The materials used and the methods adopted in this study are presented below.

2.1. Materials

The materials required in this study are listed below:

A. Battery (75 amp flooded lead acid battery)

B. DC starter motor

C. Radiator

D. Water pump

E. Coolant (ethyl glycol and water)

F. Motor fans

2.1.1. Data

The primary data were obtained from recording the traffic times from four different dense traffic spots in the city of Port Harcourt for a period of one year (2020), before the improved current traffic system in the city due to the introduction of connecting flyovers. The secondary data on fuel consumption of different engine sizes was found from publications.

2.2. Methods

In this study, a quantitative research method involving experimentation and mathematical modelling was employed. The experimentation process involved recording field data on traffic operation, while the mathematical model was required to formulate failure probability analysis of the starting and idling process. A series probability configuration is preferred, since the failure of the starting system or the cooling system-driven idling procedure will assumedly be considered as the failure of the entire system. In other words, the vehicle does not operate if the starting system fails, and if the cooling system fails, subsequent starting will be unhealthy, except the overheating is regulated, in which case the operation is stalled during the repair period.

Traffic time data are generated from the experimental investigation process of vehicular traffic operation from a traffic-congested section of a major traffic spot. A Bayesian Weibull parameter estimate method is adopted for the determination of a Weibull distribution function's shape and scale parameters in fitting the start-stop and idling systems failure data. Individual cost functions are developed for the start-stop and idling traffic driving decision for the duration of the estimated daily traffic time, in which the product of the probability of failure or the failure rate per traffic time and the replacement or repair cost for both systems and the fuel consumption costs form the component parts of any traffic drive decision. The costs analysis is done to evaluate which traffic drive decision is most economical, and effective under some prevailing operational variables.

2.2.1. Modelling of the Traffic Times

Let the average daily traffic time per month be τ_j, then given that the vehicle daily entry and exit time from the traffic is $\tau_{(e)j}$ and $\tau_{(ex)}$, per month respectively, then τ_j can be expressed as

$$\tau_j = \tau_{(ex)j} - \tau_{(e)j} \qquad (1)$$

From equation (1) the average traffic time (τ_k) per period (K) is given as;

$$\tau_k = \sum_{j=1}^{k} \frac{(\tau_{(ex)j} - \tau_{(e)j})}{K} = \sum_{j=1}^{k} \tau_j \qquad (2)$$

Where *j* is the index for the months per *K* period
Assuming the average monthly idling time is $\tau_{(I)j}$, then the *K* period idling time τ_{Ik} is;

$$\tau_{(I)k} = \sum_{j=1}^{k} \frac{\tau_{(I)j}}{K} \qquad (3)$$

Similarly, if $\tau_{(I)i}$ is the failure time for the ith idling time, then the mean failure time for n idling times is:

$$t_{(I)n} = \sum_{i=1}^{n} \frac{t_{(I)i}}{n} \qquad (4)$$

2.2.2. The Starter System Failure Time Test

Given the primary data of the initial experiment and the Subsequent derived parameters, a complete vehicle Starter System (the ignition system, a $75amp$ flooded-load acid battery, Starter motor) was set up. A number of starts operations (N) was carried out per day for stated duration ($\tau_i = \tau_1, \tau_2, \ldots, \tau_n$), at a start cycle interval of $\tau_{ic} = \tau_{1c}, \tau_{2c}, \ldots, \tau_{nc}$ as shown in fig 1.
The respective failure times for each of the time t_{si} ($t_{si} = t_{s1}, t_{s2}, t_{s3}, \ldots, t_{sn}$) are recorded are presented in table 4.2.

2.2.3. The Idling Failure Time Test

Similar to the start-stop failure time test, given the determined value of average idling time in test one, a 2010 model Toyota Camry (2.5liter displacement (4-spark plugs) and 6000rpm), with newly installed double cell radiator, functioning cooling fans and less than two weeks in service water pump is idled for $(\tau_{Ii} = \tau_{I1}, \tau_{I2}, \ldots, \tau_{In})$, times and the respective

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failure times were recorded as ($t_{Ii} = t_{I1}, t_{I2}, t_{I3}, \dots, t_{In}$), and presented in table 4.2. Note that each startup test for a given start cycle interval was done per day to relieve the starter system of the test of the previous test, which was the same approach in the idling case.

2.2.4. The Bayesian Estimation of Weibull Distribution Parameters

Supposing in the parameters estimation procedure that in a start-stop and idling test process, for $\tau_{(s)i}$ and τ_{Ii} trial times, that no failure or few failure events occurred for the entire trial times , such that almost all the tests were right censored and idling trial times τ_{Ii} are now equals to the failure times t_{Ii}, so that, $t_{Ii} = F_{Ii}+\tau_{Ii}, where\ F_{Ii}$ is the i^{th} failure event time, then the estimation of the shape parameter (β_I) and that of the scale α_I will proceed as follows;

The Weibull Survival (reliability) function $R(t_{Ii})$ is given as;

$$R(t_{Ii}) = e^{-\left(\frac{t_I}{\alpha_I}\right)^{\beta_I}} \quad for\ i = 1,2, \dots, n \qquad (5)$$

For Weibull with censoring, the likelihood is given as:

$$\ell(\beta_I, \alpha_I) = \prod_{i\epsilon F} \frac{\beta}{\alpha}\left(\frac{t_i}{\alpha}\right)^{\beta-1} \qquad (6)$$

The likelihood $\ell(\beta_I, \alpha_I)$ of all the right-censored n observations (tests) is given as;

$$\ell(\beta_I, \alpha_I) = \prod_{i=1}^{n} R(t_{Ii}) = e^{-\sum_{i=1}^{n}\left(\frac{t_{Ii}}{\alpha_I}\right)^{\beta_I}} \qquad (7)$$

The log likelihood

$$\ell = \ell(\beta_I, \alpha_I) = -\sum_{i=1}^{n}\left(\frac{t_{Ii}}{\alpha_I}\right)^{\beta_I} \qquad (8)$$

Let the prior probabilities (β_I, α_I) parameters β_I and α_I be such that

$$\beta_I \sim G(a, b) \qquad (9)$$
$$\alpha_I \sim G(a, b)\ - \qquad (10)$$

Where $G(a, b)$ is an independent Gamma function.

Hence for the case of $a = 1\ and\ b = 1$ (weak prior) for ease of computation we have that the density on the prior

$$\pi(\beta_I) = e^{-\beta_I} \qquad (11)$$

and

$$\pi(\alpha_I) = e^{-\alpha_I} \qquad (12)$$

Where $\pi(\beta_I)$ and $\pi(\alpha_I)$ are the probability density function of the prior probability.

Given $\pi(\beta_I)$ and $\pi(\alpha_I)$, the unnormalized posterior $P^*(\beta_I, \alpha_I)$ becomes

$$P^*(\beta_I, \alpha_I) = \pi(\beta_I)\pi(\alpha_I).e^{\ell} \qquad (13)$$

Where ℓ is the log-likelihood.

Hence, given equation (11) and (12), equation (13) becomes

$$P^*(\beta_I, \alpha_I) = e^{-\beta_I}.e^{-\alpha_I}.\sum_{i=1}^{n}\left(\frac{t_I}{\alpha_I}\right)^{\beta_I} \qquad (14)$$

Let

$$S(\beta_I) = \sum_{i=1}^{n} t_{Ii}^{\beta} \qquad (15)$$

$$P^*(\beta_I, \alpha_I) = e^{\frac{S(\beta_I)}{\alpha_I} - \beta_I - \alpha_I} \qquad (16)$$

The normalization constant Z, is a double integral of the unnormalized posterior densities given as

$$z = \int_0^{\infty}\int_0^{\infty} e^{-\left(\frac{S(\beta_I)}{\alpha_I} - \beta_I - \alpha_I\right)} d\alpha_I d\beta_I \qquad (17)$$

Equation (16) has no closed form; hence it is computed numerically (trapezoidal numerical integration)

Let $\beta_I \in [\beta_{Io}, \beta_{Im}]$ and $\alpha_I \in [\alpha_{Io}, \alpha_{Im}]$

Supposing the grid point $[\beta_{Io,}\ \beta_{Im}]$, span $[\beta_{Imin}\ ,\beta_{Imax}]$ with spacing of $\Delta\beta_I$ and that $[\alpha_{Io}, \alpha_{Im}]$, span $[\alpha_{Imin}, \alpha_{Imax}]$ with a spacing of $\Delta\alpha_I$, then Z is given as:

$$Z \approx \Delta\beta_I\Delta\alpha_I \sum_{j=0}^{M}\sum_{k=0}^{N} W_j V_k P^*(\beta_{Ik}, \alpha_{Ik}) \qquad (18)$$

for $j\ =\ 1, 2, \dots \dots\ m$
$k\ =\ 1, 2, \dots \dots\ n$

Where the trapezoidal weights are:

$$W_j = \begin{cases} \frac{1}{2} & j = 0 \; or \; j = M \\ 1 \end{cases} \quad (19)$$

$$V_k = \begin{cases} \frac{1}{2} & k = 0 \; or \; k = N \\ 1 \end{cases} \quad (20)$$

Precisely the value of Z is given as:

$$Z \approx \Delta\beta_I \Delta\alpha_I \left(\frac{1}{4} P^*{}_{0,0} + \sum_{j=1}^{M-1} \frac{1}{2} P^*{}_{1,0} + + \sum_{k=1}^{N-1} \frac{1}{2} P^*{}_{0,k} + \sum_{j=1}^{M-1} \sum_{k=1}^{N-1} P^*{}_{j,k} + \cdots \right) \quad (21)$$

Supposing the likelihood function $P^*(\beta_I, \alpha_I)$ is expressed in terms of log-likelihood $\ell(\beta_I, \alpha_I)$ to avoid underflow in computing P^*

Then,

$$\ell_{jk} = log \;\; P^*(\beta_I, \alpha_I) \quad (22)$$

$$\underline{P_{jk}} = e^{(\ell_{Jk} - \ell_{mn})} \quad (23)$$

Given equation (22) and (23), Z becomes:

$$Z = \int P = e^{,\ell_{mn}} \int e^{(\ell_{jk} - \ell_{mn})} \approx$$

$$\approx e^{\ell_{max}} \Delta\beta_I \Delta\alpha_I \sum_{j,k} W_j V_k P_{j,k} \quad (24)$$

The posterior mean $E[\beta_I]$ and $E[\alpha_I]$ are given as:

$$E[\beta_I] = \frac{\sum_{kj} W_j V_k \beta_I P_{ij}}{\sum_{j,k} W_j V_k P^*_{j,k}} \quad (25)$$

And,

$$E[\alpha_I] = \frac{\sum_{jk} W_k V_j \alpha_I P_{ij}}{\sum_{j,k} W_j V_k P_{jk}} \quad (26)$$

2.2.5. Modeling the Survival, Failure and the Hazard Rate Function of the Start-Stop and Idling Process Probability Distribution

The Weibull distribution being the analytical tool employed in this study, rightly models both an increasing and a decreasing failure rate, depending on the value of the shape parameter, where a shape parameter in the range of $0 < \beta_s < 1$ indicates a decreasing failure rate and $\beta_s > 1$ indicates and increasing failure rate.

Supposing the conventional reliability function $R(t)$ represents the probability that the start-stop driving system will survive beyond a traffic time τ, then according to Nahmias (2009), the reliability or survival function can be given as;

$$R(\tau_s) = P_r\{\underline{t} > \tau_s\} = e^{-\widehat{\alpha_s}(\tau_s)\beta_s} ; for \; \tau_s \geq 0 \quad (27)$$

The cumulative failure distribution $F(\tau_s)$ is given according to the same report above is;

$$F(\tau_s) = P_r\{\underline{t} > \tau_s\} = 1 - -e^{-\widehat{\alpha_s}(\tau_s)\beta_s} \quad for \; \tau_s \geq 0 \quad (28)$$

Where $\underline{t}$, is the operating life of the entire starter system.

The failure or hazard rate $r(\tau_s)$ is given as;

$$r(\tau_s) = \hat{\alpha}_s \hat{\beta}_s (\tau_s)^{\widehat{\beta}_s - 1} ; for \; \hat{\alpha}_s \; and \; \hat{\beta}_s > 0 \quad (29)$$

Similarly, the probability that the idling system will survive beyond a traffic time τ_I , which is the reliability or survival function is given as;

$$R(\tau_I) = P_r\{\mathrm{t} > \tau_I\} = e^{-\widehat{\alpha_I}(\tau_I)\beta_I} ; for \; \tau_I \geq 0 \quad (30)$$

The cumulative failure distribution $F(\tau)$ is given according to the same report above is;

$$F(\tau_I) = P_r\{t \leq \tau_I\} = 1 - e^{-\widehat{\alpha_I}(\tau_I)\beta_I} ; \quad for \; \tau_I \geq 0 \quad (31)$$

Where, t is the expected life of the entire idling system.

The failure or hazard rate $r(\tau_I)$ is given as;

$$r(\tau_I) = \hat{\alpha}_I \hat{\beta}_I (\tau_I)^{\widehat{\beta}_I - 1} ;for \; \hat{\alpha}_I \; and \; \hat{\beta}_I > 0 \quad (32)$$

2.2.6. The Start-Stop and Idling Driving Operations Costs Formulations

The start-stop driving operation requires that when a driver arrive a traffic queue, the car ignition is turned off, hence fuel consumption is put on halt, until when the traffic light initiate a movement command, by which time, the driver cranks the engine. Here, there is start up fuel consumption of θ_s per start operation and if the number of start up per τ traffic time is (N), then let θ_{SN} be the quantity of fuel consumed for N numbers of startups, θ_{sm} is the fuel consumed in the movement interval between start-stop or idling for τ time, ₦ω_s is the replacement or

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repair cost per $r(\tau_s)$ failure rate and $P_r(\tau_I)$ is the probability that there will be failure of the starter system within the operating time, then the total start-stop operating cost of the vehicle for time τ is given as;

$$C_{ss} = C_L[\ \theta_{SN} + \theta_{sm}] + [r(\tau)_s\omega_s]P_r(\tau)_{ss} \qquad (33)$$

Where C_L, is the cost of fuel per litre.

In the idling driving decision, when the driver arrives the beginning of the traffic queue, the vehicles come to a halt with respect to movement, but the engine idles with the entire cooling system operational.

Supposing the fuel consume for per τ traffic time on idling θ_I liters and that of the movement time is θ_{IM}, consider the failure rate of the idling system per (τ) to be $r(\tau)_I$ the repair or replacement cost per failure is ₦ρ_I and that of the probability that the idling system will fail on or before (τ) traffic time is $P_r(\tau)_I$, then the total idling cost per traffic time (τ) is given according to Festus (2025b) as:

$$C_{Is} = C_L[\ \theta_I + \theta_{IM}] + [r(\tau_I)\rho_I]P_r(\tau_I) \qquad (34)$$

The cost savings of the start-stop drive operation over the idling operation if any is given as

$$C_s = C_{Is} - C_{Ss} = C_L\{[\ \theta_I + \theta_{IM}] - [\theta_{SN} + \theta_{sm}]\}$$
$$+[r(\tau_I)_{\rho_I}]P_r(\tau_I) - [r(\tau_s)\omega_s]P_r(\tau_s)\}$$

3. RESULT AND DISCUSSION

The results from drive discussion, the calculation that follows and the subsequent discussion of the outcome are presented in the following sequel:

Table 3.1 shows the total average stop-start and idling drive traffic times data for January to December 2020 while table 3.2. shows a censored start cycle interval test for eleven start cycles and censored idling test results.

Table 3.1. Determination of the Traffic Time

Months J	Average daily traffic times, $\tau_j = \tau_{ex} - \tau_e$, (Hrs.)	Average daily idling times, τ_{Ij}, (Hrs.)	Average daily movement time, τ_m	Average daily No. of start operations, (N_s)	Remark
Jan	1.00	0.22	0.18	13.4	December holidays spill-over and school resumption
Feb	0.96	0.74	0.22	10.1	
Mar	0.91	0.772	0.19	9.6	
April	0.86	0.66	0.20	9.1	Wet season & beginning of holidays
May	0.92	0.77	0.15	8.3	
June	1.15	0.98	0.17	13.8	
July	1.44	1.28	0.16	14.5	Wet season's peak
August	1.28	1.09	0.19	10.9	
September	1.20	1.06	0.14	10.3	School resumption (1st term)
Oct	0.81	0.71	0.10	8.8	Regular traffic
Nov	1.02	0.93	0.09	9.2	Festivity build up
Dec	1.40	1.26	0.14	13.7	Festive period

Given that the number of months per period K are 12 $(K = 12months)$, then from the entries in table 3.1 and from equation (3), the daily traffic time per k period (one year) τ_k is:

$$\tau_k = 1.08\ hrs$$

The number of daily start cycles per K period (one year) from equation (4) is

$$N_k = 10.975hrs$$

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The total average daily idling time or stop period for K period τ_{Ik} or τ_{Sk} is:

$$\tau_{Ik} = 0.91\ hrs$$

The total average daily movement time per K period (one year) from equation (7) is;

$$\tau_{(m)k} = \sum_{j=1}^{k} \frac{\tau_{(m)i}}{K} = 0.16hrs$$

The average daily stop interval for a period (one year) from equation 6 is;

$$\tau_c = \frac{1.08}{11} = 0.0982 \approx 0.098hrs$$

$$= 5.88mins$$

The results for the start-up test of eleven (11) starts for given test times ($\tau_{si} = 0.1,\ 0.4,\ 0.6, 0.8, 1.0, 1.2$) with their respective start intervals [$\tau_{ic} = 0.009$, 0.036, 0.055, 0.073, 0.091, 0.109] and failure time $t_{si} = 0.009, 0.036, 0.055, 0.073, 0.091, 0.109$] for a censored test are given in table 3.2

Table 3.2. A Censored Start Cycle Interval Test for Eleven Start Cycles and Censored Idling Test Results

Test index number i	Startup test time τ_{si} (hrs.)	Start cycle interval time τ_{ic} (hrs)	Startup failure time t_{SI} (hrs.)	Event indicator or failure status δ_{si}	Idling test time τ_{Ic}	Idling failure time t_{Ii}	Failure status δ_{Ii}	No of start cycles n	Remarks
1	0.1	0.009	0.009	1	0.1	0.1	0	11	An irregular cranking sound for the start-stop system
2	0.4	0.036	0.036	0	0.4	0.4	0	11	Regular Engine cranking
3	0.6	0.055	0.055	0	0.6	0.8	0	11	
4	0.8	0.073	0.073	0	0.8	0.8	0	11	
5	1.0	0.091	0.091	0	1.0	0.87	1	11	Slight
6	1.2	0.109	0.109	0	1.2	0.97	1	11	Temperature rise above the normal engine efficiency temperature for the idling system
Σ			0.373	1		3.74	2		Same slight temperature rise noticed for the idling system

3.1. Estimates of the Failure and Right-Censored Failure Time Data

3.1.1. The Bayesian Weibull Parameters Estimates for the Right-Censored Start-Stop and Idling Failure Time Data Sets

Given that the event of failure F occurred only for the start interval of 0.009h for the 0.1hour test time, we have that the failure sample data is given as:

$$t_{si} = [F + \tau_{ic}]$$

$$F = [0.009h]\ (failure\ event\ time)$$

and

$$\tau_{ic} = [0.036, 0.055, 0.073, 0.091, 0.109]hrs$$

$$\Rightarrow\ t_{si} = [0.009, 0.036, 0.055, 0.073, 0.091, 0.109]hrs$$

$$\delta_{si} = [1, 0,0,0,0,0]$$

$$m_f = 1$$

Assuming the prior probabilities of β_s and α_s are approximated as:

$$\beta_s \sim G(1,1)$$

and

$$\alpha_s \sim G(1,1)$$

such that:

$$\beta_s \in [0.01, 10]\ (250\ grid\ points)$$
$$\Delta\beta_s = 0.025(steps)$$

and

$$\alpha_s \in [0.0001, 2.0]\ \ (for\ 250\ grid\ points)$$
$$\Delta\alpha_s = 0.015(step)$$

Bayesian inference was performed using a Weibull likelihood with right censoring, with an independent Gamma (1,1) prior on the shape and scale parameters. A posterior sampling was carried out via a metropolis-Hasting algorithm of the Markov Chain Monte Carlo (MCMC) approximation method over 60,000 iteration and a burn-in of 10,000, leaving 50,000 samples for analysis, for $\beta_s \epsilon [0.01,10]$ for 250 grid points and $\Delta\beta_s = 0.025$ and $\alpha_s = [0.0001,2]$ for 250 grid points, $\Delta\,\alpha_s = 0.015$.

The posterior estimate yielded, $\widehat{\beta_s} = 0.57, SE(\widehat{\beta_s}) = 0.34$, MCMC 95% credible interval $CI\ (\widehat{\beta_s}) = [0.18,1.36]$, $\widehat{\alpha_s} = 1.69hr$, $SE(\widehat{\alpha_s}) = 1.58hr$ and $CI(\widehat{\alpha_s}) = [0.39, 5.72]hrs$.

In the idling failure time trial test, failure occurred at the $0.87hr$ and $0.97hr$, while the remaining previous four observations remain right-censored at $(0.1,0.4,0.6,0.8)hrs$. Given a Gamma (1,1) on the prior, such that $\beta_I \sim$ (1,1) and $\alpha_I \sim$ (1,1) for the following data information: $t_{Ii} = [\tau_{Ii} + F]hr$, $F = [0.87,0.97]hr$, $\tau_{Ii} = [0.1,0.4,0.6,0.8]hr$, t_{Ii}= [0.1,0,4,0.6,0.8,0.87,0.92]hr, σ_{Ii}= [0,0,0,0,1,1], $m_I = 2, \beta_I \in$ [0.001,5] for 200 grid points at $\Delta\beta_I$ =0.025, $\alpha_I \in$ [0.001,3] for 200 grid points at $\Delta\alpha_I$=0.015 and 30,000 iteration, 8000 burn-in for 5 thinning. Bayesian estimation of Weibull shape and scale parameters for the two-failure case, using Metropolis–Hastings MCMC and right-censored data given the following problem status;

Problem setup (two-failure case)

- Test times (hours):
  - 0.87 h (during 1.0 h test)
  - 0.97 h (during 1.2 h test)
- Right-censored at:
  - 0.1, 0.4, 0.6, 0.8 hours

In the Bayesian inference carried out for the idling failure time data, the posterior estimates yielded: $\widehat{\beta_I}$ =2.1781, $SE(\widehat{\beta_I})$ = 1.35, $\widehat{\alpha_I}$ =1.194hr, SE ($\widehat{\alpha_I}$) =0.59hr. Other results include: CI ($\widehat{\beta_I}$) = [0.3138, 5.6184], RSE ($\widehat{\alpha_I}$) =49.41 and CI ($\widehat{\alpha_I}$) = [0.4217, 2.7315]

3.1.2. The Weibull Probability Distribution Evaluation with the Bayesian Parameters Estimates of the Failure and Right-Censored Failure Time Data

The Bayesian parameters estimates of the starter system for one failure and five right-censored data gave the following;

$$\hat{\beta}_s = 0.57, \qquad \hat{\alpha}_s = 1.69$$

Hence, the probability that the start-stop system will survive beyond 1.08hrs traffic time of eleven (11) start-ups of 0.0982hrs start interval is:

$$R(0.0982) = 0.6375 = 63.75\%\ chances$$

Hence, the probability of failure for eleven (11) start-ups for a start-up interval of 0.09827hr is:

$$\text{F}(0.0982) = 0.3625\ \ or\ \ 36.25\%\ chances$$

The number of failures per eleven start-ups of 0.982hr start-up interval is:

$$r(0.0982) = 0.9633(0.0982)^{-0.43}$$
$$= 2.61\ failure\ per\ 0.0982hrs\ of\ startup\ interval$$

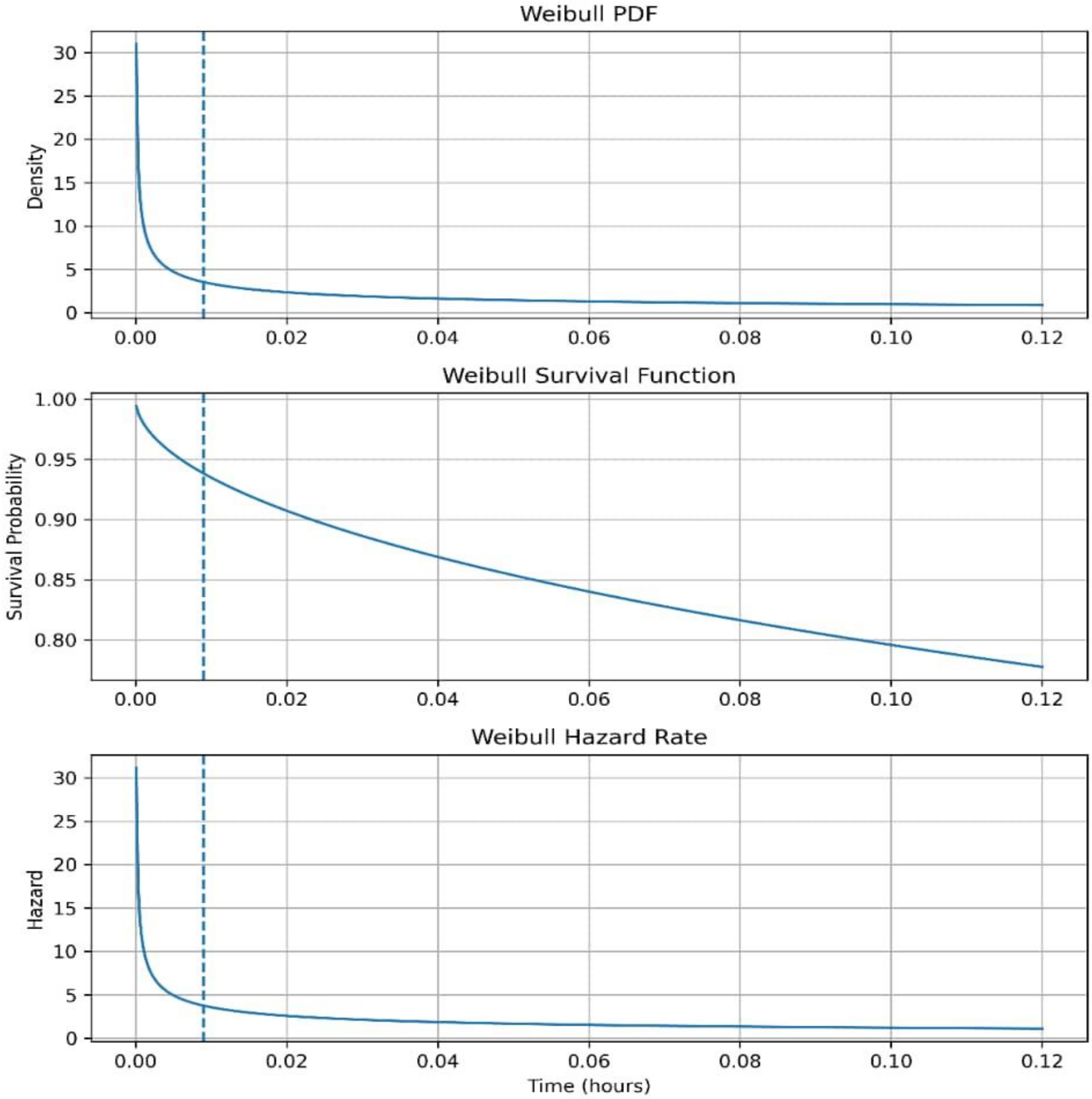


Figure 1: The Weibull PDF (a), Survival Function (b) and Hazard rate (c) of the Start-Stop Failure time Distribution for the one Failure Case under Bayesian Estimation

Similarly, given that the Bayesian parameter estimate for the idling system of two failures with four right-censored data gave the following data:

$$\hat{\beta}_I = 2.1781, \qquad \hat{\alpha}_I = 1.1942hr$$

Hence, the probability that the idling system will survive beyond 0.91hrs traffic idling time is:

$$R(0.91) = 0.3782 \quad 0r \ \ 37.82\% \ chances$$

The cumulative failure F(τ_I) is:

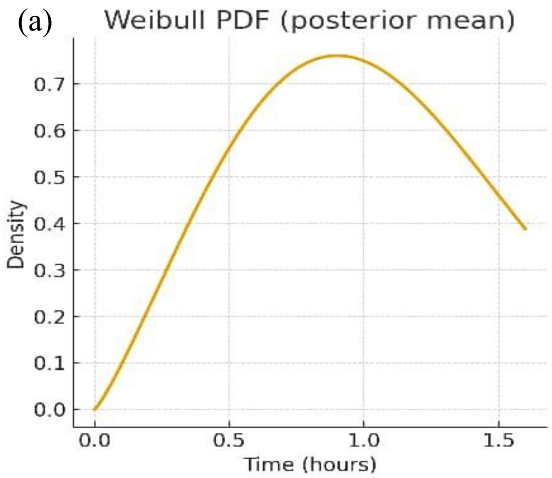
$$F(\tau_I) = 0.6218 \ \ or \ \ 62.18\% \ chances$$

The hazard rate $r(\tau_I)$ is given as:

$$r(0.91) = 2.3 \ failues \ per \ 0.91h \ traffic \ iddling \ time.$$

Estimation for the Weibull lifetime distribution of the idling system are shown below:

(a)



(b

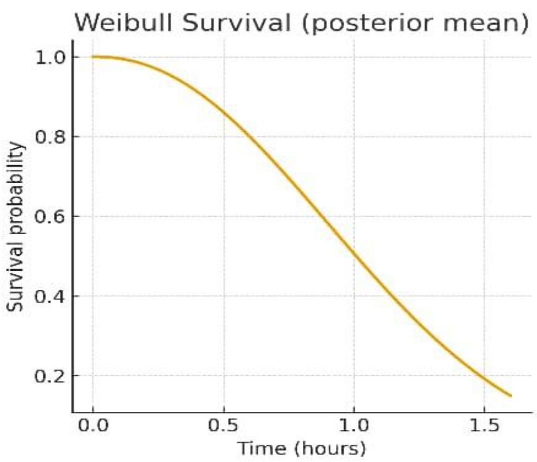


Figure 2: The Probability Density Function (PDF), (a) and the survival plots for the Bayesian parameter (b)

3.1.3. The Start-Stop and Idling Costs Evaluation Based on the Bayesian Parameter Estimate for Failure Plus Right-Censored Data.

Given that are costs components values remain constant, i.e.;

$$C_L = ₦900.00\ (cost\ per\ litre\ of\ fuel)$$
$$\omega_s = ₦70{,}000$$
$$\theta_{SN} = 0.0132L$$
$$\theta_{sm} = 0.4L$$

Since the start-stop system operation cost is c_{ss}

Then $C_{ss} = ₦66{,}600.62$

For the idling operation cost, given:

$$\rho_s = ₦70{,}000$$
$$\theta_I = 2.275L$$
$$\theta_{MI} = 0.4L$$
$$\Rightarrow C_{Is} = ₦115{,}573.1$$

The cost savings in favour of the start-stop system $(C_{Is} - C_{ss})$ is given as:

$$C_{Is} - C_{ss} = C_s = ₦48{,}974.48\ \ or\ \ 42.38\%.$$

3.1.4. Bayesian Weibull Parameter Estimates for All Right-Censored Start-Stop and Idling Failure Time Data Analysis

Assuming both the starter and idling systems were technically sound (no history of failure) as the results of the tests case almost suggested if the weak reasons for failure were ignored, such that all the failures were right -censored, then given the following data, the Bayesian analysis will give:

Supposing out of the six (6) trials of the start-stop driving test of eleven (11) start-ups at varying intervals τ_{sI}; the event of failure did not occur such that all the tests were right censored at their test interval duration, then the tests startup interval times τ_{ic} are taken as the failure times.

Hence,

$$t_{si} = [0.009, 0.036, 0.055, 0.073, 0.091, 0.109]$$
$$n = 6$$

From equation 32, the failure event indicator δ_{si} is;

$$\delta_{si} = [0,0,0,0,0,0]$$
$$M_f = 0$$

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Given $\beta_s \in [0.01,10]$ *and* $\alpha_s \in [10^{-4}, 2.0]$ *[250 grid point]*

Bayesian inference yielded:

$Z = 0.2916$, $E[\beta_s] = 1.75$, and $E[\alpha_s] = 0.92$

Hence $\hat{\beta}_s = 1.75$, and $\hat{\alpha}_s = 0.92$

Similarly, for the idling, assuming that

$$\beta_I \sim G(1,1)$$

And

$$\alpha_I \sim G(1,1)$$

Where $G(1,1)$ means the independent gamma approximation of the scale parameter

Let the grid points on the shape and scale parameter be:

$\beta_I \in [0.01,5]$ (200 grid points)

$$\Delta\beta_I = 0.025$$

And,

$\alpha_I \in [0.001,3]$ (200 grid points)

$$\Delta\alpha_I = 0.015$$

$t_{Ii} = [0.1, 0.4, 0.6, 0.8, 1.0, 1.2]$ hours

$\beta_I \approx 0.0013,\ \alpha_I \approx 0.2043$ and $Z \approx 0.03465$

Given that $\hat{\beta}_s = 1.75$, and $\hat{\alpha}_s = 0.92hr$

The probability that the starter system will survive eleven (11) start-ups of start intervals $\tau_c = 0.0982hr$ for a right censoring time test for $\tau_c \in t$ is;

$$R(0.0982) = 0.9843 = 98.43\%$$

$$F(0.0982) = 1 - 0.9843 = 0.0157 = 1.57\%$$

$$r(0.0982) = 1.61(0.098)^{0.75} = 0.282$$

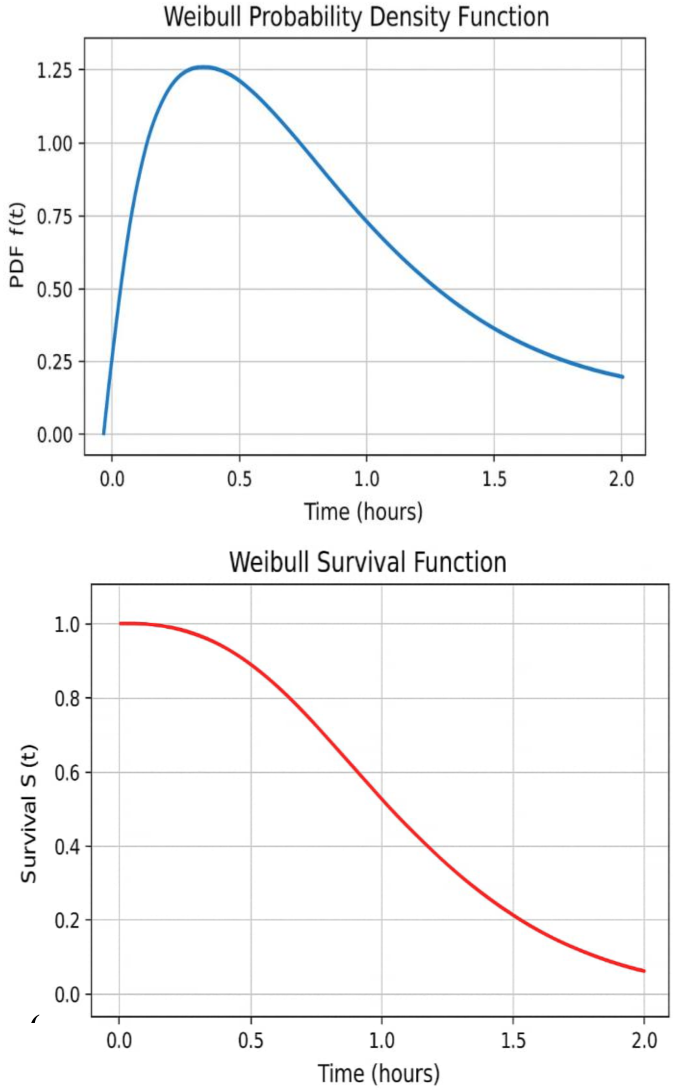

Figure 3: Weibull Probability Density Function (a) and the Survival Function (b) for Right Censored Start-up Interval Failure Times

$$\beta_I = 0.0013 \text{ and } \hat{\alpha}_I = 0.2043$$

The probability that a vehicle will survive traffic idling time of $\tau_I = 0.91hr$ for $\tau_I \in t_I$ is;

$$R(0.91) = 0.8153 = 81.53\%$$

$$F(0.91) = 1 - R(0.91) = 1 - 0.8153$$

$$= 0.1847 = 18.47\%$$

$$r(0.91) = 0.000266(0.91)^{-0.9987} = 0.00029$$

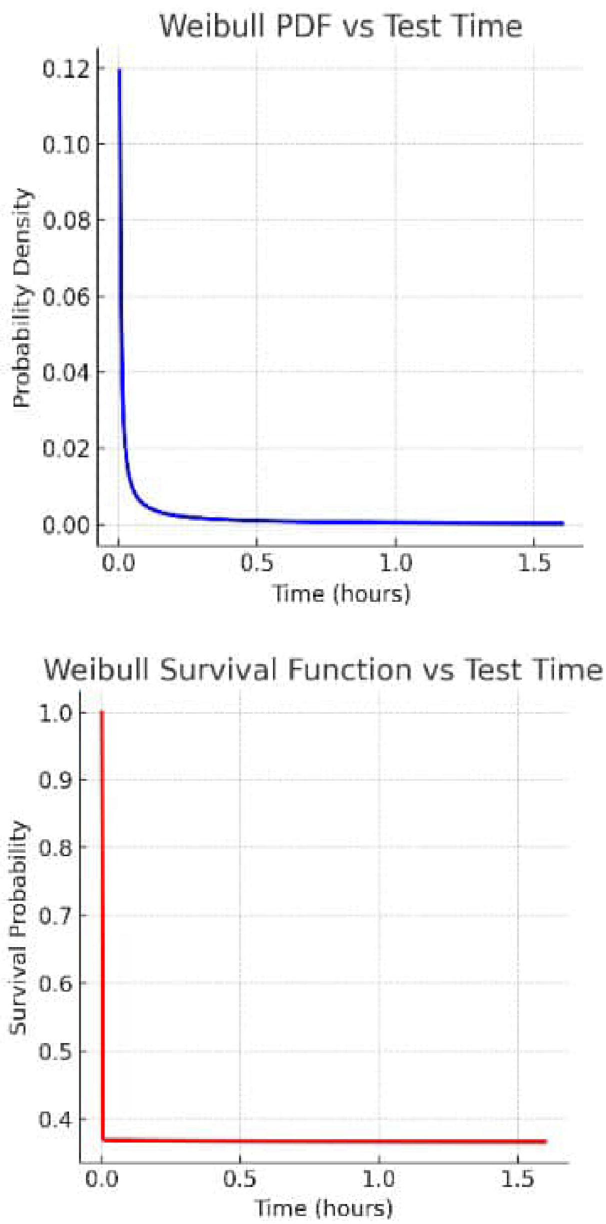

Figure 4: The Weibull Failure (a) and Survival Probability (b) for Right Censored Idling Failure Times

The start-stop driving operation costs from the censored time test is

$$C_{ss} = \#681.80$$

The idling operation costs for the time censored test is

$$C_{Is} = \#2{,}411.25$$

The cost savings of the start-stop idling decision over that idling for the time censored test C_s is given as;

$$C_S = C_{Is} - C_{Ss} = \#1{,}729.45 = 71.72\%$$

3.2. Discussion

Using a Gamma (1,1) prior on the Weibull parameters for the one failure start-stop failure data (a supposedly starter system that has been in operation), the Bayesian inference yielded posterior mean estimates of : $\hat{\beta}_s$= 0.57 , SE ($\hat{\beta}_s$) = 0.34, $\hat{\alpha}_s$= 1.69hr,and SE(, $\hat{\alpha}_s$)= 1.58hr The probability that the start-stop system will survive 0.0982hr start interval for 11starts is R(0.0982) = 0.6375 or 63.75% chances, that it will fail F(0.0982) = 0.3625 or 36.25% chances and the failure rate r(0.0982) = 2.61 failures per eleven starts of 0.0982hr start interval for 1.08hrs of traffic time.

For the Bayesian parameter estimates approach to the two failure and four right-censored idling failure sample data(the same technical status for the above starter system) ,the determined shape and scale parameters estimates are : $\hat{\beta}_I \approx$ 2.18 and $\hat{\alpha}_I \approx$ 1.1942hours respectively, the standard errors are :SE($\hat{\beta}_I$) = 1.35 and SE($\hat{\alpha}_I$) = 0.59. The probability that the idling system will fail on or before 0.91 hours under the Bayesian estimation is $F(0.91) = 0.6218$ or 62.18% chances, that it will survive 0.91hours of idling is $R(0.91) = 0.3782$ or 37.82% chances and the hazard rate is r(0.91) = 2.3 failures per 0.91 hours idling time. For the all right-censored start-stop failure time data, the Bayesian inference yielded posterior mean estimates of : $\hat{\beta}_s = 1.75$, SE($\hat{\beta}_s$) = 0.62, RSE($\hat{\beta}_s$) = 35.43% and CI($\hat{\beta}_s$) = [0.78,3.15], $\hat{\alpha}_s$ =0.92hr, SE($\hat{\alpha}_s$) = 0.41, RSE($\hat{\alpha}_s$) = 44.57%, CI ($\hat{\alpha}_s$) = [0.32,1.75] , a normalization constant $Z = 0.2916$, with a reliability of R(0.0982)

=0.9843 or 98.43% chances, cumulative failure of F(0.0982) =0.0157 or 1.57% chances and failure rate of r(0.0982) =0.282. In the case of the all right-censored idling data, Bayesian inference yielded estimated shape $(\widehat{\beta_I})$ and scale $(\widehat{\alpha_I})$ parameters of: 0.0013 and 0.2043 respectively and a normalization constant (Z) of 0.03465 for the no failure events.The survival and failure probabilities and the failure rate for the all right- censored idling traffic drive decision for 0.91hr idling time respectively are;R(0.91) = 0.8153 or 81.53% chances,F(0.91) = 0.1847or 18.47% chances and r(0.91) = 0.000292 failures per 0.91hr idling time. Unlike the failure plus right- censored start-stop case, the right censored idling operation of no failure event resulted in less reliability (81.53% against 89.57%) of the idling drive decision, even though it reported a much smaller failure rate of 0.00029 against 2.3 of the failure event case.

The probability density function (PDF) curve of fig.1a for the Bayesian parameter estimate for the Weibull distribution shows a strong peak at the beginning which is in tandem with infant-mortality behaviour, that of fig.1b shows a rapid drop in the survival distribution with a long tail indicative of the censored data and the hazard function of fig.1c is decreasing since the shape parameter is less than one confirming early- life failure prevalence. The (PDF) curve of fig.2a rise to a peak between 0.9-1.1h which matches the recorded failures at 0.87hr and 0.97hr and starts declining after 1.2hr, meaning it is less likely that the idling system will fail after the expected life region. The survival function of fig.2b shows a survival probability that is high at the beginning of the curve but decline sharply between 0.87hr and 0.97hr. Summarily, the idling system is stable initially degrades rapidly after about 0.87hr with most failures occurring within approximately 1hr, and displaying increasing wear-out behaviour rather than random failure. The PDF curve of fig.3a is distributed across time, it starts from zero since no early failures were seen and increases slightly before tapering off. The survival function of fig.3b on the other hand start from 1 and decays smoothly with time, meaning reliability decreases with increasing time. The PDF of all right -censored idling data of fig.4a is almost zero for all times because the shape parameter is extremely low (0.0013), making the curve almost flat, indicating failure is almost impossible at any time within the test time of 1.2hrs, hence the hazard rate is decreasing. The survival curve is close to 1 and it barely declines since no failure was observed. The Bayesian starter system operating cost estimates for the failure plus right-censoring are: C_{ss} = ₦66,600.62, that of the idling cost operation C_{Is} = ₦ 115,573.1, with start-stop cost savings of C_S = ₦ 48,974.48 or 42.38%. The operating costs for the start-up system for the all right -censored failure data set gives ; C_{ss} = #681.80 and that of the idling operation is C_{Is} = #2,411.25 , with a cost savings of C_S = #1,729.45 (71.72%) of the start-stop system over that of idling resulting from the idling system higher failure probability and fuel consumption cost.

3.3. Findings

In the start-stop driving system for the failure plus right-censored data (implying a system with considerable service time) were the entire starter system has failure history, it was found that failure rate decreased with time, hence as the starter system is more endangered with smaller start-up intervals. On the other hand, the idling system showed a failure rate that increased with time; hence the idling system is more failure prone with prolong idling times. In both cases, this is the trend in the real-life situation, were more frequent start-ups and prolong idling time make both drive systems susceptible to failure. In terms of the technical analysis with respect to which of the driving decision is more reliable when both systems are fairly healthy due to prong service with possible repair history, the start-stop system showed 63.75% chances of survival against 36.25% chances of the idling system, even though their failure rates are quite close, with that of the idling lower with 11.88%. Hence, with aging effect of both systems, the start-stop decision is more preferable for traffic time within the range of one hour and above since its failure rate decreases with time and its increasing reliability with time. Economically, the start-stop drive operation had a savings of #48,974.48 or 42.38% over that of the idling, majorly for the idling system high fuelling and repair costs. The no failure event of the all right-censored data did not allow the parameter estimation method to learn about failure of the systems, hence the lifetime distribution for both systems negated the normal trends o decreasing failure rate with time for the start-stop operation and an increasing failure rate for the idling system.

4. CONCLUSION

In this study, an in-depth research into traffic drive decision has been conducted and results based on technical analysis have been tendered for which the most cost-efficient driving decision could be made depending on the nature of the traffic situation and on the technical status of the starter or idling systems. The age long preference for a particular traffic drive decision-based on emotional or presumptive inclination have been corrected. Even though there may be some elements of experimental and mathematical confidence issues for which the report of this study should be treated with caution, especially for the latter test cases of all right-censored data, it gives a sound footing for future research and slims the chances of guess work in the preferred choice for a traffic start-stop or idling drive decision.

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